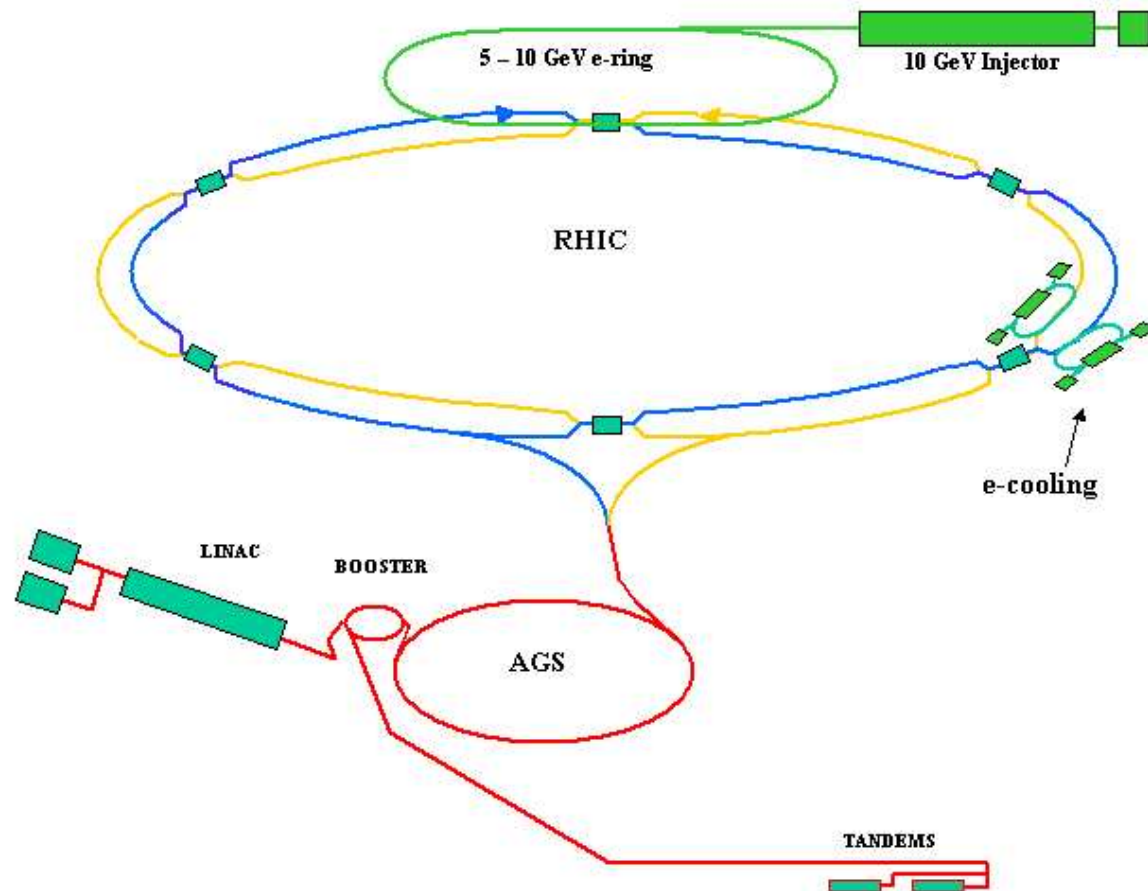


Beam-beam simulation studies for eRHIC

Christoph Montag

C-AD seminar, September 9, 2005

eRHIC layout



IR parameters for 10 GeV e on 250 GeV p

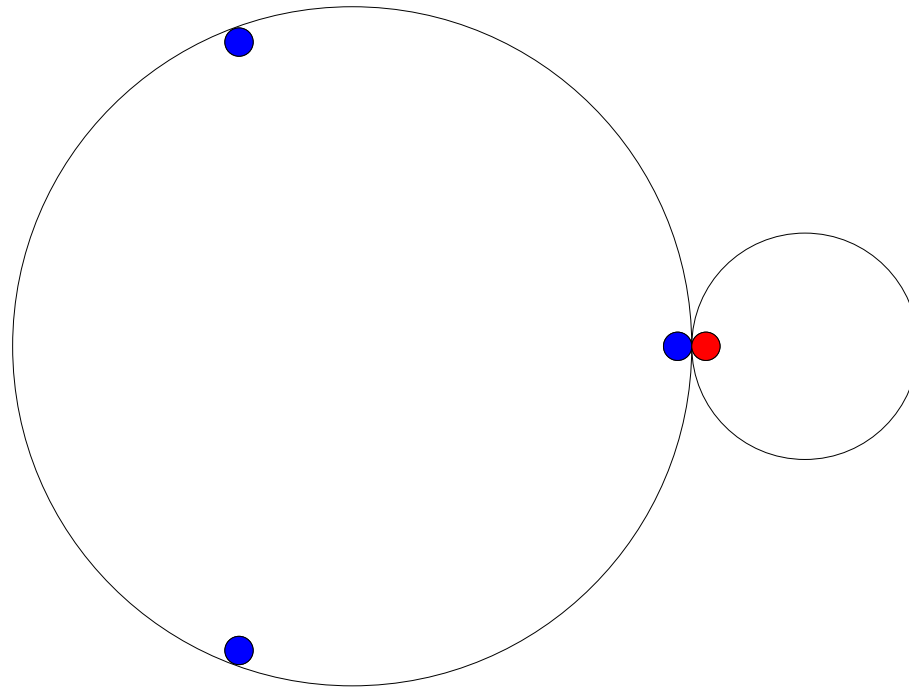
C_p [m]	3834
C_e [m]	1278
ϵ_p [nm]	9.5
ϵ_e (x/y) [nm]	53/9.5
β_p (x/y) [m]	1.08/0.27
β_e (x/y) [m]	0.19/0.27
σ^* (x/y) [μm]	100/50
σ_s [mm]	11.7
Q_s	0.04
τ (x/y/s) [turns]	1740/1740/870
N_e/bunch [10^{11}]	1.0
N_p/bunch [10^{11}]	1.0
ξ_p (x/y)	0.007/0.0035
ξ_e (x/y)	0.022/0.08
\mathcal{L} [$\text{cm}^{-2}\text{sec}^{-1}$]	$4.4 \cdot 10^{32}$

Areas of concern

- Unequal circumferences
- Optimum working point to support large electron beam-beam tunes shift
- Non-Gaussian transverse electron beam tails

Unequal circumferences

Transverse barycenter motion due to unequal circumferences (Hirata and Keil, NIM A 292 (1990), 156 – 168)

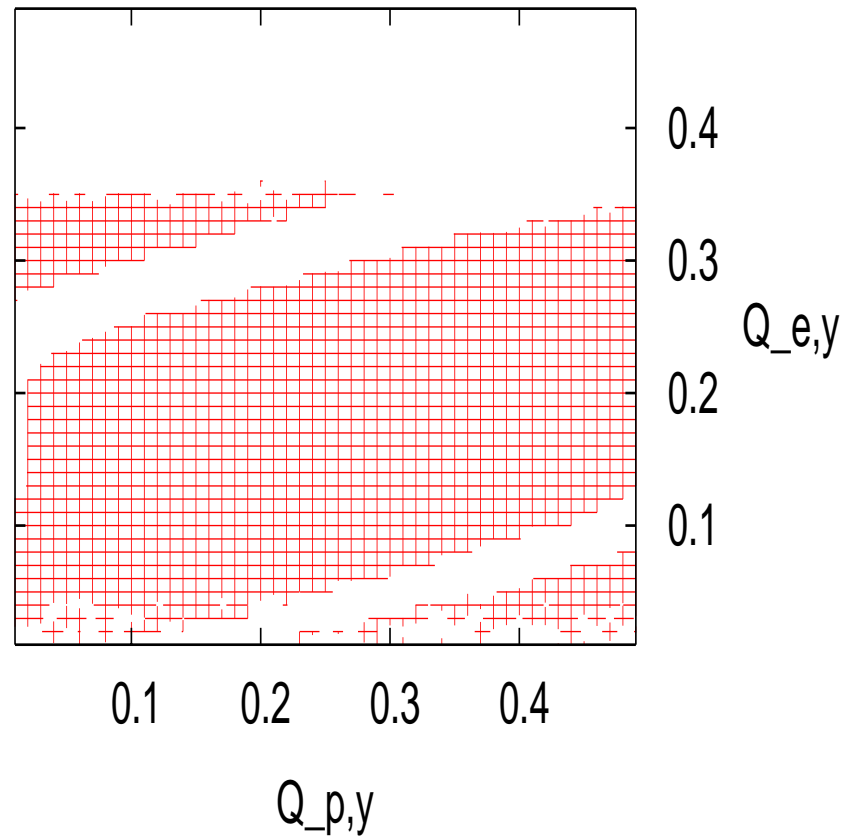


Resonance condition: $Q_p - 3 \cdot Q_e = n$,
resonance width to be determined by simulation

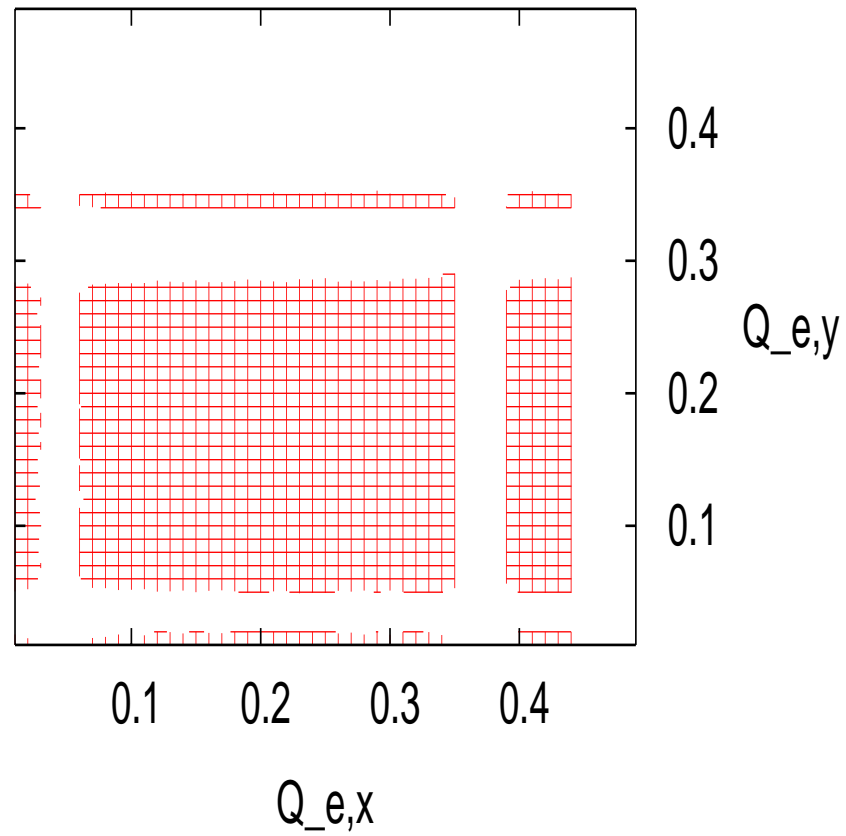
Simulation technique

- track one electron bunch, interacting with three proton bunches
- each bunch is represented by the centroid particle
- starting with a tiny offset ($1\text{ }\mu\text{m}$) in both planes, observe betatron amplitude (action) over 10000 RHIC turns
- perform electron beam tune scan (fixed RHIC tunes)

Beam-beam resonances in the vertical plane



Beam-beam resonances for proton working point (.21,.23)



Working point search

Strong beam-beam lens modifies the entire lattice:

- Beam-beam tune shift

$$\cos(2\pi Q) = \cos(2\pi Q_0) - 2\pi\xi_0 \sin(2\pi Q_0)$$

- Dynamic β^*

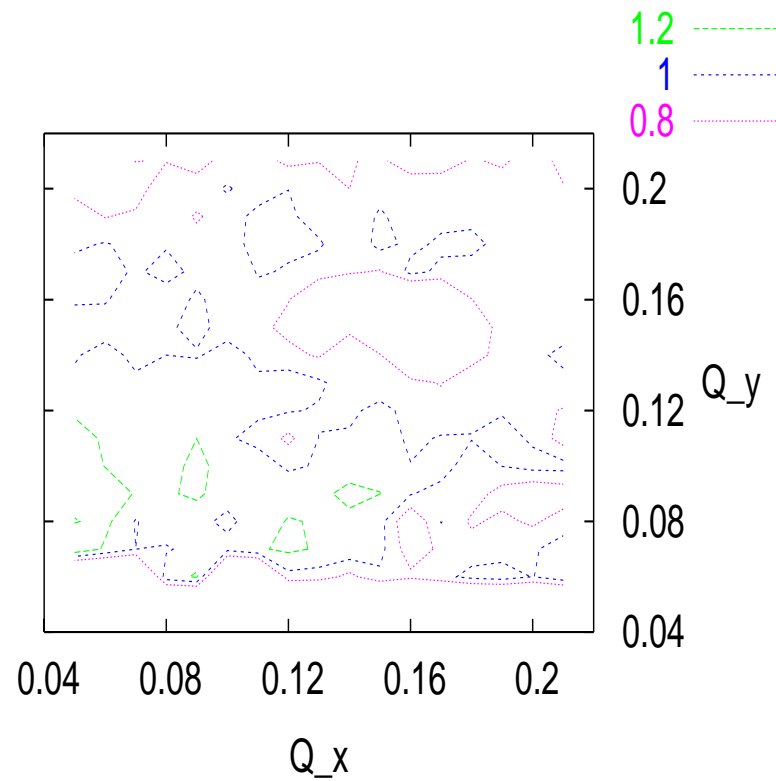
$$\beta^* = \beta_0^* \frac{\sin(2\pi Q_0)}{\sin(2\pi Q)} = \frac{\beta_0^*}{\sqrt{1 + 4\pi\xi_0 \cot(2\pi Q_0) - 4\pi^2\xi_0^2}}$$

- Induced β -wave modifies \mathcal{H} and therefore synchrotron integrals (equilibrium emittance, damping times)

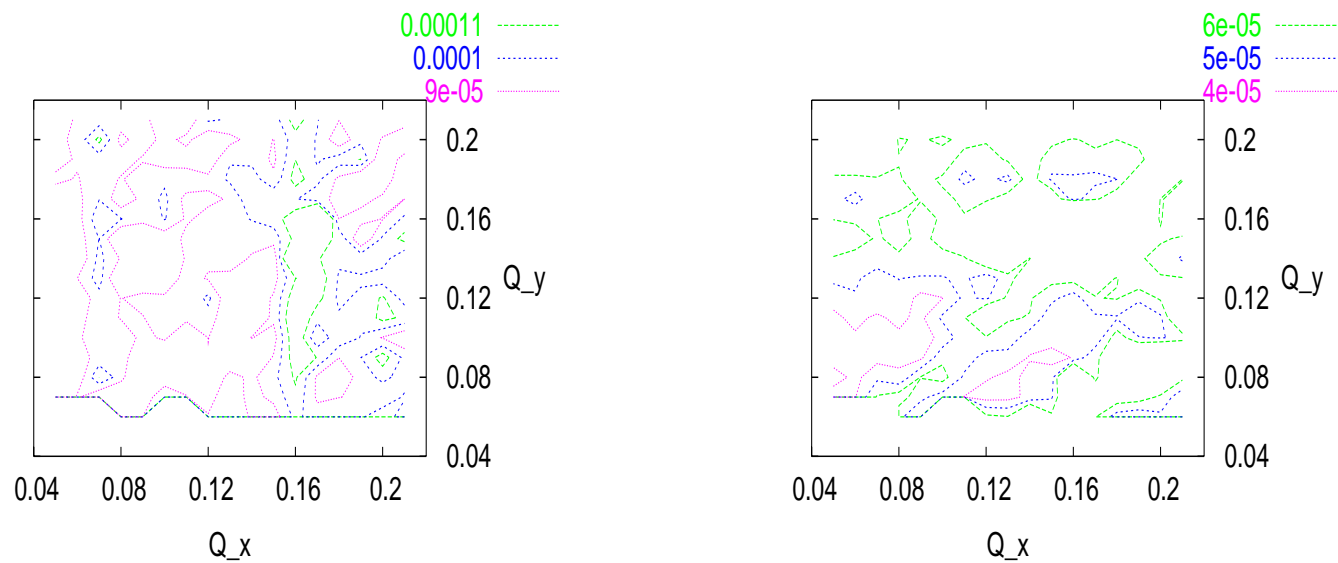
Simulation technique

- 6D-tracking of 1000 particles through non-linear lattice (element-by-element), including synchrotron radiation, for ten radiation damping times
- radiation integrals for each working point are calculated by MAD; these determine equilibrium emittance for linearized beam-beam kick
- equilibrium beam sizes and therefore luminosity are calculated by averaging over final 500 turns of tracking

Luminosity for $\xi_y = .08$, in units of design luminosity



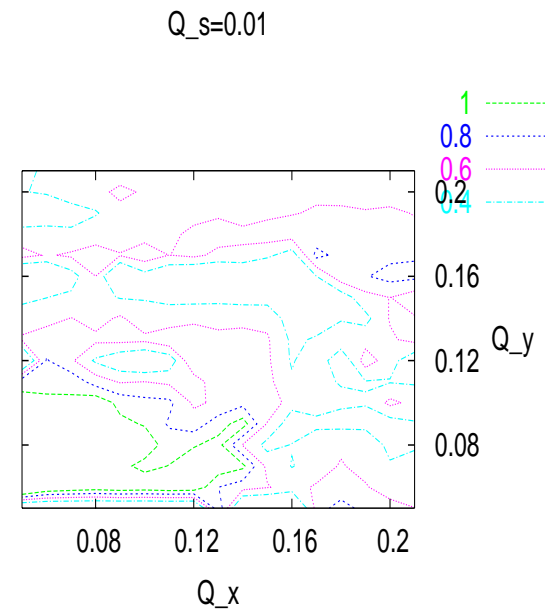
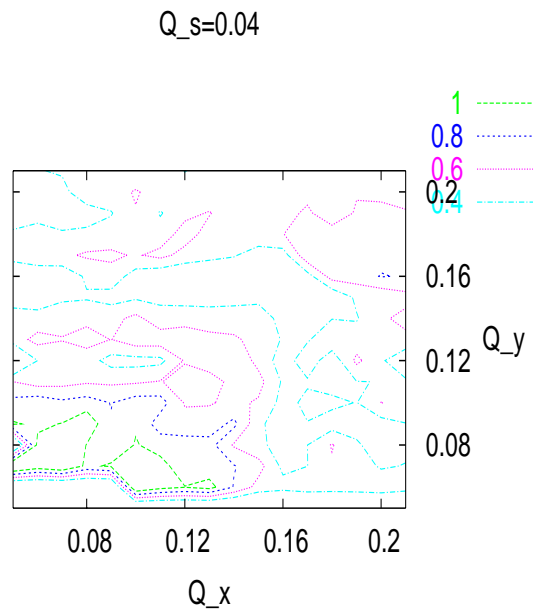
Equilibrium beam sizes for $\xi_y = .08$



Working point candidates:

$$(Q_x, Q_y) = (.10, .14), (.05, .07), (.14, .07)$$

Luminosity for $\xi_y = .16$, for different synchrotron frequencies



Higher tunes shift may be feasible, but may require lower synchrotron frequency

→ Different lattice (non-FODO)

Transverse beam tails

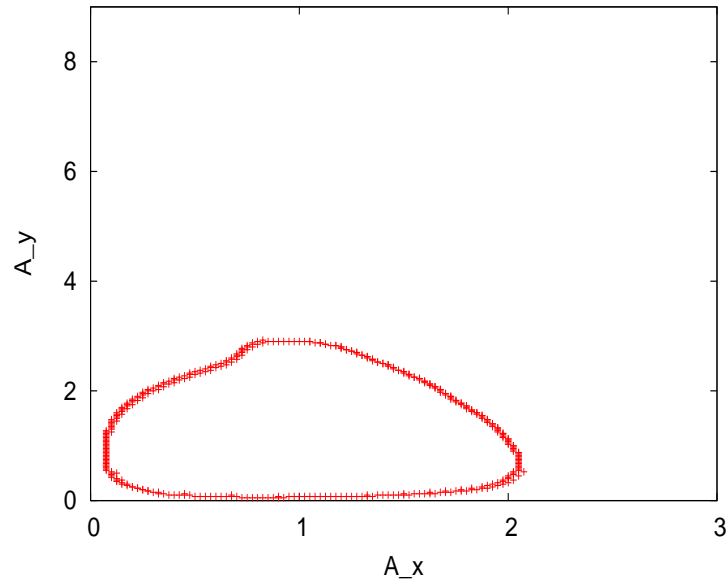
Enhanced, non-Gaussian transverse tail population is a concern

- Beam lifetime
- Synchrotron radiation background in detector from particles in transverse tails

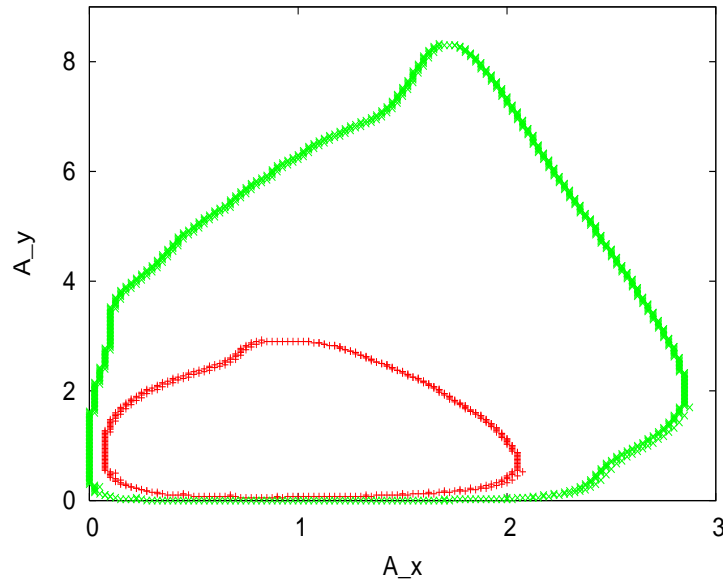
Simulation technique

Independently developed by D. Shatilov in Novosibirsk (Part. Acc. 52, pp. 65 – 93) and T. Chen, J. Irwin, and R. Siemann at SLAC (PRE 49, **3**, pp. 2323 – 2330)

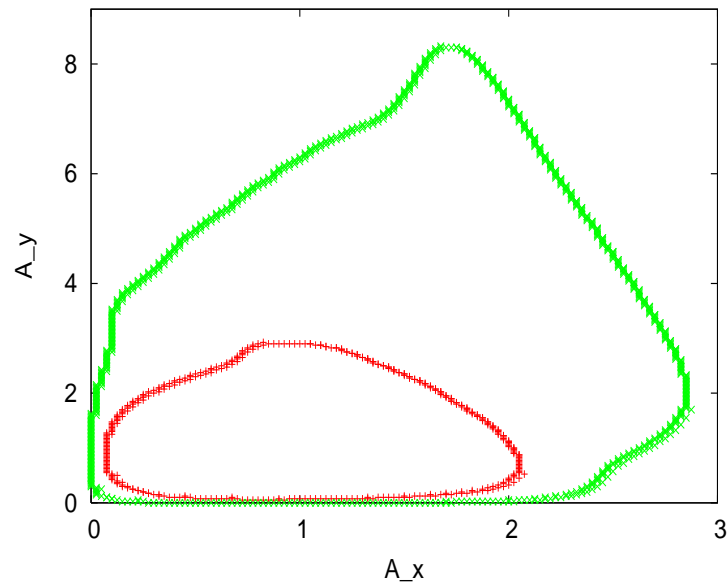
6D-tracking of a single particle over 10000 damping times for each step, linear one-turn matrix plus beam-beam and synchrotron radiation



1. **a.** establish density distribution in normalized amplitude space ($A_x = x/\sigma_x, A_y = y/\sigma_y$) during 10000 damping times
- b.** determine border where density has dropped by factor k with respect to maximum at $(A_x, A_y) = (1, 1)$
- c** track another 10000 damping times; save coordinates whenever the particle crosses the border to the “outside”



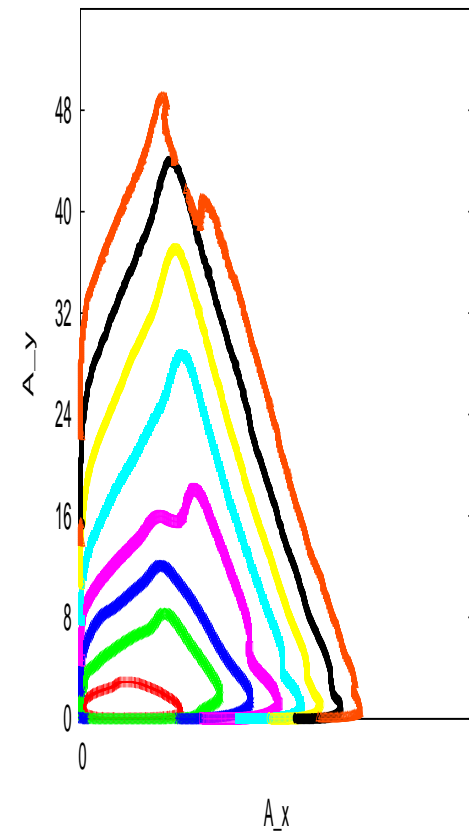
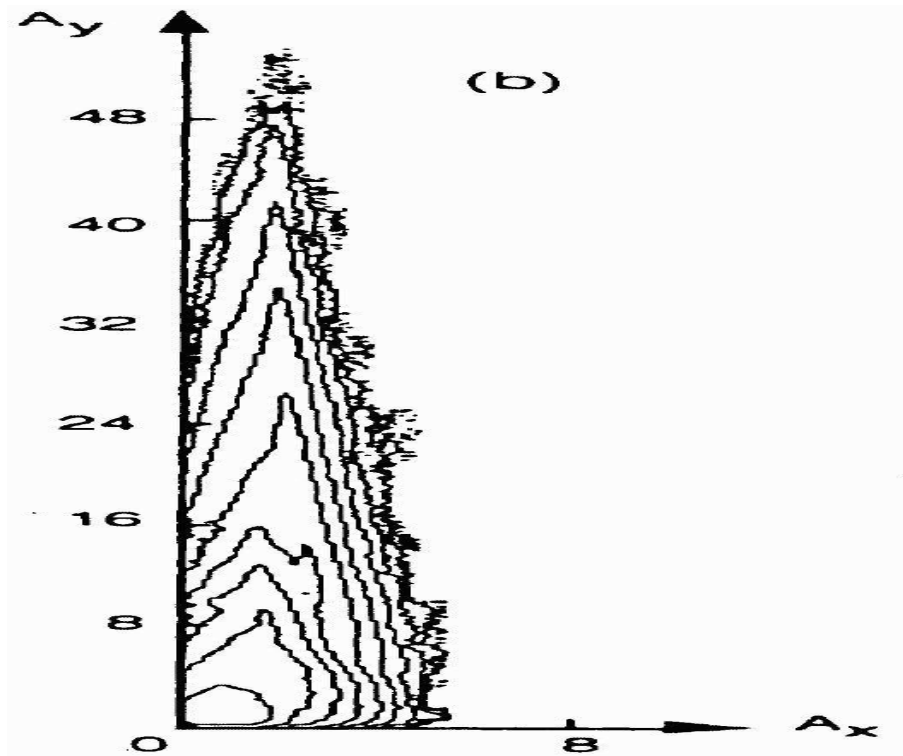
2. **a.** start tracking at one of the previously saved coordinates; whenever particle falls below the border, re-insert at randomly chosen set of previously saved coordinates
- b.** establish density distribution in amplitude space
- c.** determine border where density has dropped by a factor k with respect to inner border



d. track again; re-insert if particle crosses inner border; save coordinates if particle crosses newly established outer border

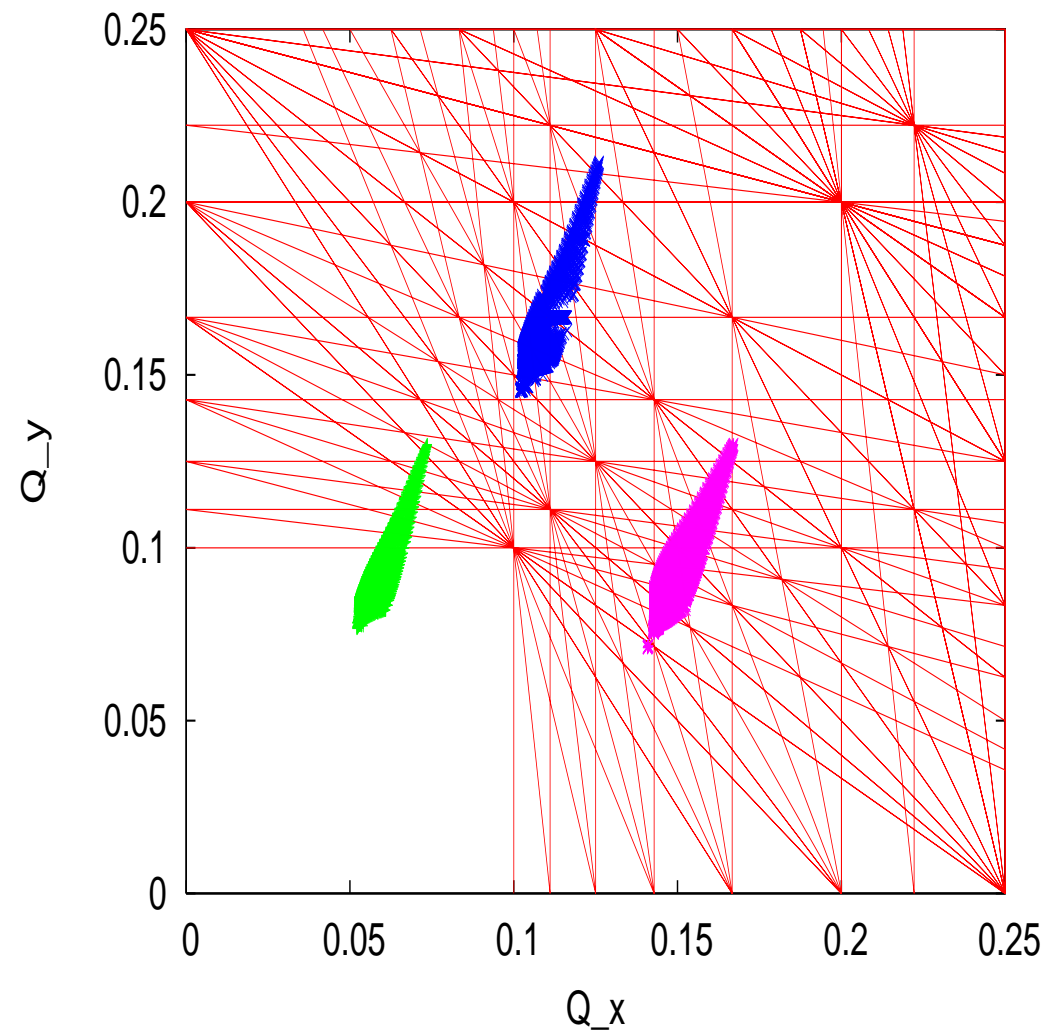
3. iterate, starting from step 2.

Benchmarking with PEP-II parameters (T. Chen et al.)

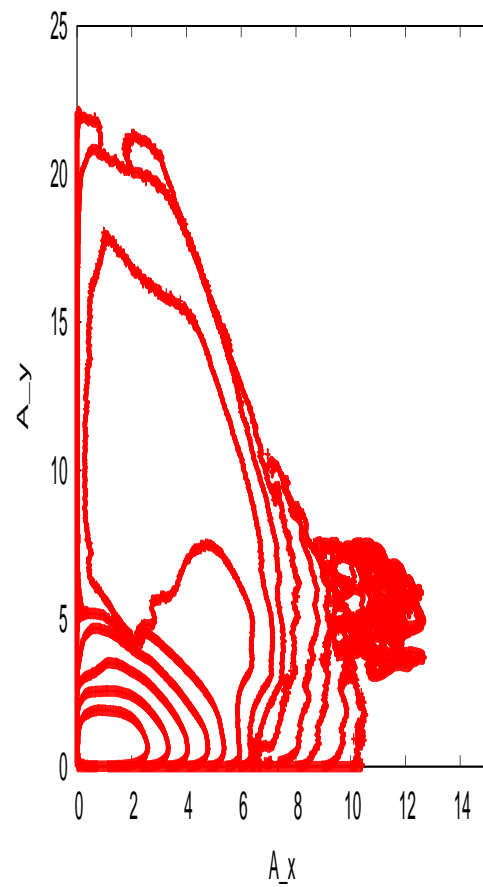
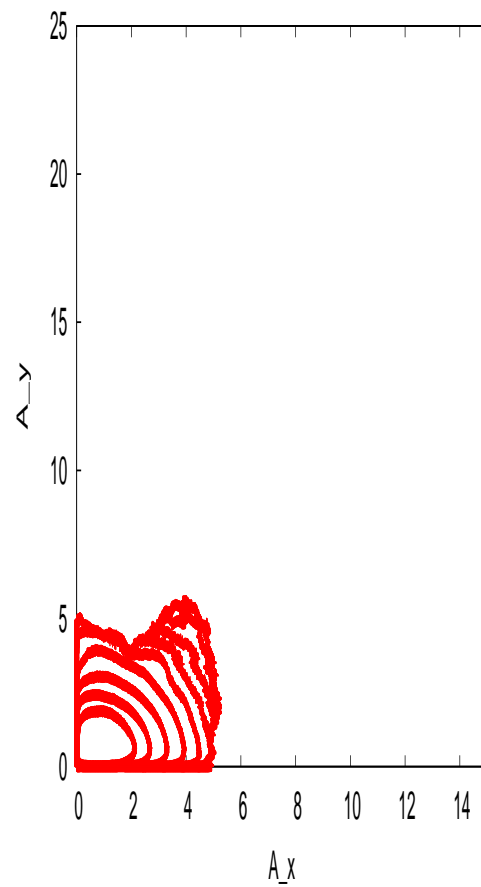
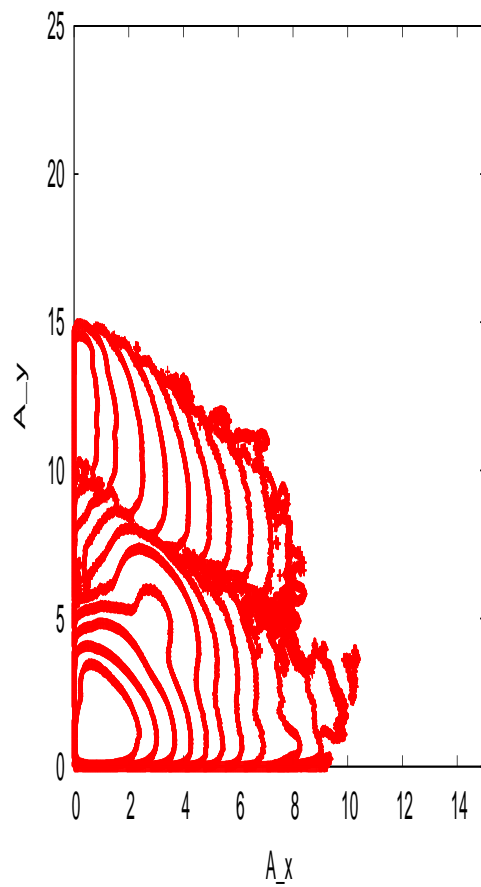


Vertical tail is caused by extremely flat beams at IP,
 $\sigma_x/\sigma_y = 100$

Tune footprints for three possible working points



Intensity contours for
 $(Q_x, Q_y) = (.10, .14), (.05, .07), (.14, .07)$



What causes these distributions?

Large beam-beam parameter $\xi_y = 0.08$ makes it impossible to avoid low-order resonances.

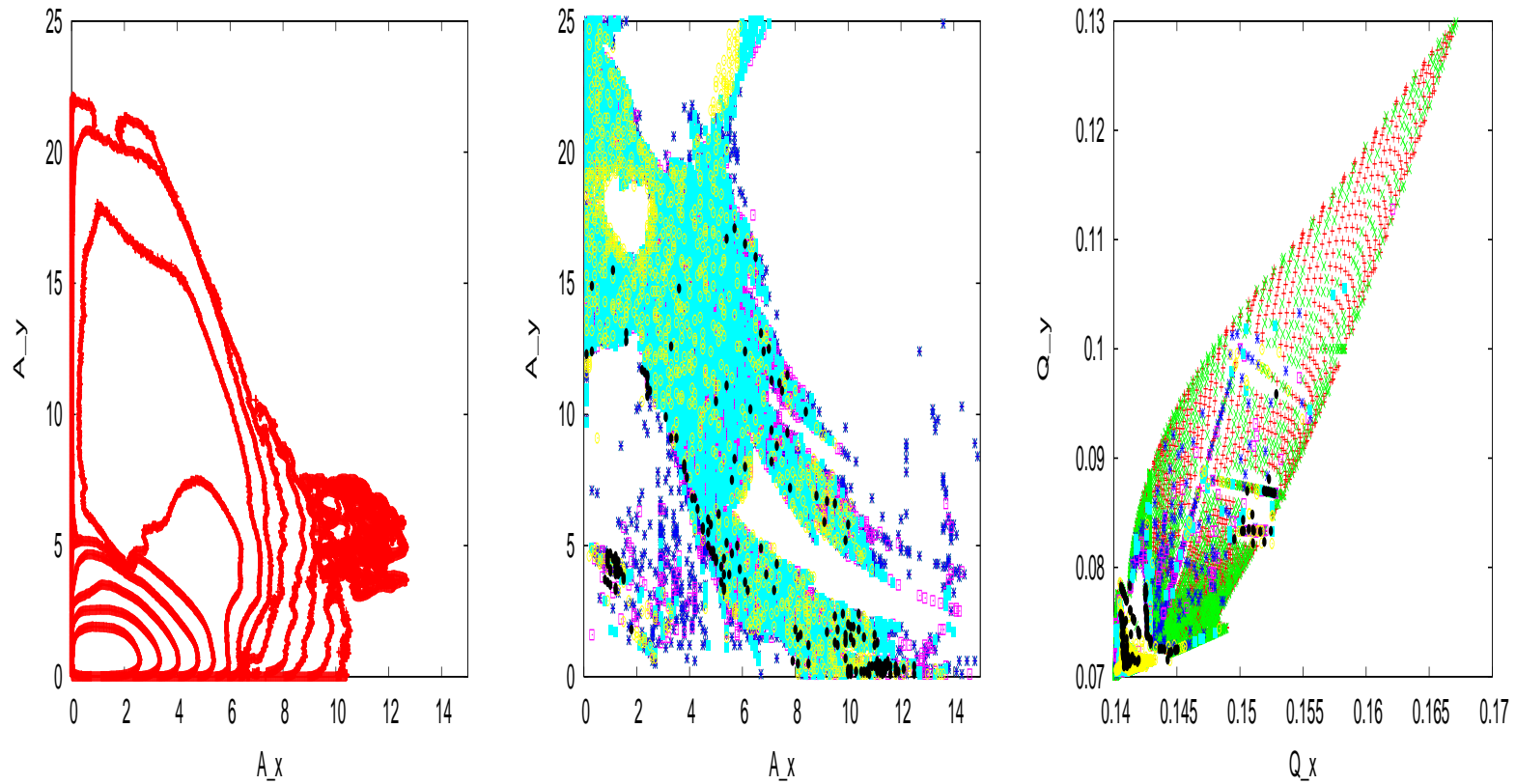
But which resonances are responsible for these transverse distributions?

Use frequency map analysis to find out (J. Laskar, *Icarus* **88**, 266 - 291 (1990))!

Frequency map analysis - how does it work?

- Launch a single particle at transverse amplitudes (A_x, A_y)
- Track for N turns (no synchrotron radiation)
- Determine working point $(Q_{x,1}, Q_{y,1})$ from FFT
- Track for another N turns
- Again, determine working point $(Q_{x,2}, Q_{y,2})$
- Plot $\Delta = \log \left(\sqrt{(Q_{x,1} - Q_{x,2})^2 + (Q_{y,1} - Q_{y,2})^2} \right)$ vs. (A_x, A_y)

Resonances in amplitude and tune space, (.14,.07)



Strongest resonances (black dots):

$$6Q_x + 2Q_y = 1, \quad 7Q_x = 1, \quad Q_x - 2Q_y = 0$$

Conclusion

- Unequal circumferences not a concern if tunes in both rings are chosen properly
- Beam-beam parameter of $\xi_y = 0.08$ (eRHIC design) can be achieved; there may even be considerable headroom
- Only moderate development of non-Gaussian tails; even high-order resonances contribute to transverse distribution